Baryon Magnetic Moments in a Simultaneous Expansion in $1/N$ and m_s

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Abstract

We consider the baryon octet and decuplet magnetic moments in a simultaneous expansion in m_s and $1/N$ taking $N_F/N \sim 1$, where N is the number of QCD colors and N_F is the number of light quark flavors. At leading order in this expansion, the magnetic moments obey the non-relativistic quark-model relations. We compute corrections to these relations using an effective lagrangian formalism which respects chiral symmetry to all orders in the $1/N$ expansion. Including corrections up to order $m_s^{1/2}$, we find 8 relations among the 9 measured octet and decuplet magnetic moments; including corrections up to order $1/N$ and m_s, we find 4 remaining relations. The relations work well, and suggest that the expansion is under control. We give predictions for the unmeasured decuplet magnetic moments.

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1. Introduction

In this paper, we consider the baryon magnetic moments in a simultaneous expansion in m_s and $1/N$, where N is the number of QCD colors. There has been a recent revival of interest in the $1/N$ expansion for baryons, started by the results of ref. [1]. For example, it was shown that many interesting large-N relations have corrections starting at $\sim 1/N^2$. These results have been extended using a number of different methods [2][3][4][5]. We will use the formalism of ref. [4]. This formalism is based on an exact relativistic treatment of QCD, yet makes direct contact with the static quark model. On a more practical level, it allows us to write an explicit effective lagrangian in which chiral symmetry is kept manifest to all orders in $1/N$.

When the number of light flavors $N_F > 2$, the $SU(N_F)$ flavor representations of baryons grow with N, and so there are ambiguities in how to extrapolate physical baryon states to $N > 3$. We use the approach of ref. [5], where it is shown that the $1/N$ expansion can be formulated to include all of the states in the $SU(N_F)$ flavor representations for arbitrary N and N_F. The physical results for $N = 3$ can be obtained without having to identify the physical baryon states with particular states for $N > 3$, and $SU(N_F)$ flavor symmetry is also kept manifest in this approach. This expansion is well-defined even if $N_F/N \sim 1$ [5]; we will work in this limit, since $N_F = N = 3$ in the real world.

In the large-N limit, the magnetic moments obey the non-relativistic quark model relations [6]. We find that the leading corrections to these results are suppressed relative to the leading terms by order $1/N$, $m_s^{1/2}$, and m_s. We will assume that $O(m_s)$ and $O(1/N)$ corrections are both $O(\epsilon) \sim 30\%$, and carry out the expansion consistently to $O(\epsilon)$. Including the $O(m_s^{1/2}) = O(\epsilon^{1/2})$ corrections, we find 8 relations among the 9 measured magnetic moments for the octet and decuplet baryons; including corrections up to $O(\epsilon)$, we find 4 surviving relations. These relations agree well with data, providing evidence that the combined $1/N$ and chiral expansions work well for baryons. We then predict the unmeasured decuplet magnetic moments including all corrections up to $O(\epsilon)$; these may be measured at CEBAF.

2. Formalism

In this section, we briefly review the formalism we will use to obtain our results. In ref. [4], it was shown how to write an effective lagrangian describing the low-energy interactions of baryons with the pseudo-Nambu–Goldstone bosons (PNGB's) for large N. The PNGB's are described in the standard way: the field

$$\xi(x) = e^{i\Pi(x)/f}, \tag{1}$$

is taken to transform under $SU(N_F)_L \times SU(N_F)_R$ as

$$\xi \mapsto L\xi U^\dagger = U\xi R^\dagger, \tag{2}$$

where this equation implicitly defines U as a function of L, R, and ξ. For $N_F = 3$, the meson fields are

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \tag{3}$$

Note that the η' is not light if $N_F/N \sim 1$, and is therefore not included. The effective lagrangian is most conveniently written in terms of the hermitian fields

$$V_\mu \equiv \frac{i}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right), \qquad A_\mu \equiv \frac{i}{2}\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right), \tag{4}$$

which transform under $SU(N_F)_L \times SU(N_F)_R$ as

$$V_\mu \mapsto UV_\mu U^\dagger + iU\partial_\mu U^\dagger, \qquad A_\mu \mapsto UA_\mu U^\dagger. \tag{5}$$

We can incorporate $SU(N_F)$ breaking due to $m_s \neq 0$ by including the quark mass spurion (for arbitrary N_F)

$$m_q \equiv m_s S, \quad S \equiv \begin{pmatrix} 0 & & & \\ & \ddots & & \\ & & 0 & \\ & & & 1 \end{pmatrix}, \tag{6}$$

transforming under $SU(N_F)_L \times SU(N_F)_R$ as $m_q \mapsto Lm_q R^\dagger$. We find it convenient to define the even-parity field

$$m \equiv \frac{1}{2}(\xi^\dagger m_q \xi + \text{h.c.}) \mapsto UmU^\dagger. \tag{7}$$

In this notation, the leading terms giving rise to meson interactions are

$$\mathcal{L} = f^2 \operatorname{tr}(A^\mu A_\mu) + \beta f^3 \operatorname{tr}(m) + \cdots, \tag{8}$$

where β is a coupling and $f = f_\pi \simeq 93$ MeV; $f \sim \sqrt{N}$ in the large-N limit [7].

We now discuss the baryon fields. Because the baryon mass is of order $N\Lambda_{\text{QCD}}$, we can describe the baryons using a heavy-particle effective field theory [8]. We write the baryon momentum as $P = M_0 v + k$, where $M_0 \sim N$ is a baryon mass and v is a 4-velocity

($v^2 = 1$) which defines the baryon rest frame. We then write an effective field theory in terms of baryon fields whose momentum modes are the residual momenta k. This effective field theory gives an expansion in $1/M_0$ around the static limit.

For N large, the baryon $SU(N_F)$ representations are large, and it is convenient to use a compact notation to keep track of baryon flavor quantum numbers. We use a Fock-space notation in which the baryons fields are written

$$|\mathcal{B}(x)) \equiv \mathcal{B}^{a_1\alpha_1\cdots a_N\alpha_N}(x)\,\alpha^\dagger_{a_1\alpha_1}\cdots\alpha^\dagger_{a_N\alpha_N}|0). \tag{9}$$

The $\alpha^\dagger$'s are *bosonic* creation operators which create a "quark" with definite flavor and spin, and $|0)$ is the Fock "vacuum" state; $a_1,\ldots,a_N$ are $SU(N_F)$ flavor indices and $\alpha_1,\ldots,\alpha_N = \uparrow,\downarrow$ are spin indices in the rest frame defined by v.

Under $SU(N_F)_L \times SU(N_F)_R$, the baryon fields transform as

$$\mathcal{B}^{a_1\alpha_1\cdots a_N\alpha_N} \mapsto U^{a_1}{}_{b_1}\cdots U^{a_N}{}_{b_N}\mathcal{B}^{b_1\alpha_1\cdots b_N\alpha_N}, \tag{10}$$

where U is defined in eq. (2). $\mathcal{B}^{\cdots}$ transforms under a highly reducible representation of $SU(N_F)$, but we will have to carry out calculations explicitly only for $N = N_F = 3$.

Meson–baryon interactions are written in terms of operators constructed from the creation and annihilation operators. For example, the leading terms involving baryons can be written

$$\mathcal{L} = (\mathcal{B}|iv^\mu\nabla_\mu|\mathcal{B}) + g\,(\mathcal{B}|\{A^\mu\sigma_\mu\}|\mathcal{B}) + \cdots, \tag{11}$$

where $\sigma_\mu \equiv (\slashed{v}\gamma_\mu - v_\mu)\gamma_5$ is the spin matrix and we define

$$\{A^\mu\sigma_\mu\} \equiv \alpha^\dagger_{a\alpha}(A^\mu)^a{}_b(\sigma_\mu)^\alpha{}_\beta\alpha^{b\beta}, \tag{12}$$

etc. The chiral covariant derivative acting on the baryon fields is defined by

$$\nabla_\mu|\mathcal{B}) \equiv (\partial_\mu - i\{V_\mu\})|\mathcal{B}). \tag{13}$$

The coupling g can be determined from matrix elements of the $\Delta S = 1$ axial current measured in semileptonic hyperon decays. We obtain $g = 0.83 \pm 0.08$, where the error is obtained by assigning a 30% uncertainty to the higher-order corrections. This value should be used with caution, since the $SU(N_F)$-breaking corrections are known to be large [8][9].

In this notation, the leading N dependence of an arbitrary term in the effective lagrangian is given by associating a factor of $1/N^{r-1}$ with every r-body operator (that is, an operator constructed from r creation and r annihilation operators), and a factor of $1/N$ for every explicit flavor trace [4]. The reason for these rules is that an r-body operator

can arise only from quark-level diagrams involving at least $r - 1$ gluon exchanges, and flavor traces arise from quark loops. Each gluon exchange or quark loop gives rise to a suppression of $1/N$, yielding the rules given above. For more details, see ref. [4]. According to these rules, the coupling g in eq. (11) is order 1 in the large-N limit.

3. Magnetic Moments

We now apply the formalism discussed in the previous section to the magnetic moments.

3.1. Leading Order

At leading order in the large-N and chiral limits, the baryon magnetic moments are described by a single term in the effective lagrangian:

$$\delta\mathcal{L} = \frac{a_0 e}{\Lambda}\, v_\mu \widetilde{F}^{\mu\nu}(\mathcal{B}|\{\mathcal{Q}\sigma_\nu\}|\mathcal{B}), \tag{14}$$

where $\widetilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$, $\epsilon_{0123} = +1$, and

$$\mathcal{Q} \equiv \frac{1}{2}(\xi^\dagger \mathcal{Q}_L \xi^\dagger + \xi \mathcal{Q}_R \xi) \mapsto U\mathcal{Q}U^\dagger. \tag{15}$$

Here (at $N_F = 3$)

$$\mathcal{Q}_L = \mathcal{Q}_R \equiv \begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{pmatrix} \tag{16}$$

are the left- and right-handed quark charge spurions. The parameter $\Lambda \sim 1$ GeV is the chiral expansion scale; naïve dimensional analysis leads us to expect that $a_0 \sim 1$. The magnetic moments arising from eq. (14) are given by

$$\mu_{B'B}\, \sigma^j_{B'B} = -\frac{a_0 e}{\Lambda}\, (B'|\{\mathcal{Q}\sigma^j\}|B), \tag{17}$$

where $\sigma^j_{B'B}$ is the matrix element of the spin matrix between the states $|B)$ and $|B')$ normalized so that its maximal value is $+1$. (We use non-script capital letters to refer to specific baryon states.) The operator $\{\mathcal{Q}\sigma^3\}$ has matrix elements $O(N)$, so that the leading contributions to the magnetic moments are $O(N)$.*

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* We could normalize the electric charge of the quarks to be of order $1/N$ so that the baryons have electric charge of order 1 in the large-N limit. If we did this, the magnetic moments would be $O(1)$ in the large-N limit. Such a rescaling would modify the formulas which follow in a trivial way, and would not affect our results for $N = 3$.

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At this order, there are 9 measured octet and decuplet baryon magnetic moments determined by a single unknown constant a_0/Λ. There are therefore 8 relations among the magnetic moments. Of these, 6 are the Coleman–Glashow relations [10]

$$
\begin{aligned}
\mu_p - \mu_{\Sigma^+} &= 0, \quad (15\%) \\
\mu_n + \mu_p + \mu_{\Sigma^-} &= 0, \quad (10\%) \\
\mu_n - 2\mu_\Lambda &= 0, \quad (40\%) \\
\mu_{\Sigma^-} - \mu_{\Xi^-} &= 0, \quad (55\%) \\
\mu_n - \mu_{\Xi^0} &= 0, \quad (40\%) \\
\sqrt{3}\mu_n + 2\mu_{\Sigma^0\Lambda} &= 0, \quad (5\%)
\end{aligned}
\tag{18}
$$

which hold in the limit of exact $SU(3)$ independent of the $1/N$ expansion; the remaining relations can be taken to be the quark-model relations

$$
3\mu_n + 2\mu_p = 0, \quad (3\%)
\tag{19}
$$

$$
\mu_{\Omega^-} + \mu_p = 0, \quad (35\%)
\tag{20}
$$

which are consequences of the large-N limit. The numerical accuracy indicated is defined by dividing the numerical value by the average of the positive and negative terms on the left-hand side. If we perform a best fit, the average deviation is 0.3 μ_N.

3.2. $1/N$ Corrections

The $O(1/N)$ corrections to the magnetic moments arise from the term

$$
\delta\mathcal{L} = \frac{a_1 e}{N\Lambda} v_\mu \widetilde{F}^{\mu\nu} (\mathcal{B}|\{\mathcal{Q}\}\{\sigma_\nu\}|\mathcal{B}).
\tag{21}
$$

Because matrix elements of the operator $\{\mathcal{Q}\}\{\sigma^\mu\}$ can be $O(N)$, the contributions to the magnetic moments arising from this term are $O(1)$ in the large-N limit. Including this term, the Coleman–Glashow relations in eq. (18) still hold, but the quark-model relations eqs. (19) and (20) no longer hold. Note that the relation eq. (19) involves states with zero strangeness, and therefore receives further corrections only from isospin breaking, which are expected to be about 5%. The experimental deviation of this relation is therefore a direct measure of the $1/N$ corrections, and we have no understanding of why the these corrections are so small in this case.

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3.3. $SU(N_F)$-breaking Corrections

In chiral perturbation theory, the leading $SU(N_F)$-breaking corrections generally arise from loop graphs with PNGB intermediate states. Such graphs can give nonanalytic dependence on the quark masses. In the case of the magnetic moments, the leading dependence on m_s is $\sim m_s^{1/2}$ and $\sim m_s \ln m_s$ [11]. The diagrams which give rise to these corrections are shown in fig. 1.

The graph which gives rise to the $\sim m_s^{1/2}$ corrections is easily evaluated using the meson–baryon coupling from eq. (11):

$$\text{fig. 1a} = -\frac{g^2 e}{2f^2} \left(B' | \{T_A \sigma^\nu\}\{T_B \sigma^\lambda\} | B \right) \text{tr}(T_A[T_B, \mathcal{Q}])$$

$$\times \int \frac{d^4 k}{(2\pi)^4} \frac{(2k+q)_\mu k_\nu (k+q)_\lambda}{(k^2 - M_A^2 + i0^+)((k+q)^2 - M_B^2 + i0^+)(k \cdot v + i0^+)}, \tag{22}$$

where the T_A are $SU(N_F)$ generators normalized so that $\text{tr}(T_A T_B) = \delta_{AB}$, and

$$M_{AB}^2 = \frac{\beta f}{2} \text{tr}(\{T_A, T_B\} m_q) \tag{23}$$

is the mass-squared matrix of the PNGB's (see eq. (8)). We have neglected the $O(m_s)$ mass differences between baryons in the same $SU(N_F)$ multiplet, as well as the $O(1/N)$ mass differences between octet and decuplet baryons. (Including these effects gives corrections suppressed by m_s and/or $1/N$.) The sum over all intermediate spin states is then included in eq. (22), with the large-N relations properly taken into account. Evaluating eq. (22) gives rise to a contribution to the magnetic moments

$$\delta \mu_{B'B} \sigma_{B'B}^j = \frac{g^2 e M_K}{16\pi f^2} (B' | \mathcal{O}_K^j | B), \tag{24}$$

where

$$\mathcal{O}_K^j = (N + N_F) \text{tr}[Q(1-S)] \{S\sigma^j\} - \{S\}\{\mathcal{Q}\sigma^j\} + \{\mathcal{Q}\}\{S\sigma^j\}. \tag{25}$$

This gives contributions to the magnetic moments which are $O(N m_s^{1/2}) = O(N \epsilon^{1/2})$.

Including these contributions along with the leading term in eq. (14), we obtain 8 relations valid to $O(N \epsilon^{1/2})$ which are independent of g. One of these is the quark-model relation eq. (19). The remaining relations can be written

$$\mu_{\Sigma^-} + \mu_n + \mu_p = 0, \quad (10\%)$$

$$\mu_{\Xi^0} - 2\mu_{\Xi^-} = 0, \quad (4\%)$$

$$\mu_{\Sigma^+} - 2\sqrt{3}\mu_{\Sigma^0 \Lambda} + \mu_p = 0, \quad (7\%)$$

$$\mu_{\Sigma^+} + \frac{1}{\sqrt{3}}\mu_{\Sigma^0 \Lambda} + \mu_\Lambda + \mu_p = 0, \quad (2\%)$$

$$\mu_{\Xi^-} + \mu_{\Sigma^+} - \mu_{\Sigma^-} - \mu_p = 0, \quad (4\%)$$

$$\mu_{\Omega^-} - \mu_{\Xi^0} - \mu_{\Xi^-} = 0. \quad (2\%)$$

$$\tag{26}$$

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In addition, there is one relation which depends on g,

$$\mu_{\Sigma^-} - \mu_{\Xi^-} = \frac{g^2 e M_K}{8\pi f^2} \qquad (-0.51 = -2.0). \tag{27}$$

(We use $f = f_K \simeq 114$ MeV in the evaluation.) The relations which are independent of g work much better than the g-dependent relations: a fit including the $O(Nm_s^{1/2})$ corrections and treating g as a free parameter has an average deviation of 0.08 μ_N. The nonanalytic corrections have the right sign, but their predicted magnitude for the lowest-order value $g \simeq 0.8$ is too large. However, we expect that including the $SU(N_F)$-breaking corrections in the fit to the semi-leptonic decays will substantially *decrease* g [8][9], and it is not clear to us that the large discrepancy in eq. (27) indicates a breakdown of the expansion.

The vertex graph (fig. 1b) and wavefunction graphs (fig. 1c) combine to give the contribution

$$\text{figs. 1b, c} = \frac{ieg^2}{4f^2} \epsilon_{\mu\nu\lambda\rho} v^\nu q^\lambda \left(B'|\left[\{T_A\sigma^\alpha\}, \left[\{T_A\sigma^\beta\}, \{\mathcal{Q}\sigma^\rho\}\right]\right]|B\right)$$
$$\times \int \frac{d^4k}{(2\pi)^4} \frac{k_\alpha k_\beta}{(k^2 - M_A^2 + i0^+)(k\cdot v + i0^+)((k-q)\cdot v + i0^+)}. \tag{28}$$

The double commutator can be written as a 1-body operator; this contribution is therefore $\sim 1/N$ times a 1-body operator, and can be at most $O(m_s \ln m_s) = O(N\epsilon^2 \ln \epsilon)$. This is negligible compared to the $O(N\epsilon)$ counterterms which we will discuss below. The graph in fig. 1d gives a contribution which is $1/N$ times a 1-body operator, which is negligible for the same reason as for eq. (28); the graph in fig. 1e gives no contribution to the magnetic moments

The leading $SU(N_F)$-violating counterterms are

$$\delta\mathcal{L} = ev_\mu \widetilde{F}^{\mu\nu}(\mathcal{B}|\left[\frac{b_1}{\Lambda^2}\{(\mathcal{Q}m)\sigma_\nu\} + \frac{b_2}{N\Lambda^2}\{\mathcal{Q}\sigma_\nu\}\{m\} + \frac{b_3}{N\Lambda^2}\{\mathcal{Q}\}\{m\sigma_\nu\}\right]|\mathcal{B}). \tag{29}$$

These counterterms give $O(Nm_s) = O(N\epsilon)$ contributions to the magnetic moments. There are also $O(Nm_s)$ contributions from the loop graph in fig. 1a, but these have the same spin–flavor dependence as the counterterms considered above, so we will not need to evaluate them explicitly. Including the counterterms in eq. (29) and the nonanalytic corrections computed above, we obtain the relations

$$\mu_{\Xi^0} + 2\mu_{\Sigma^+} + 2\mu_{\Sigma^-} + \mu_n = 0, \quad (10\%)$$
$$\mu_{\Xi^-} + 4\mu_{\Sigma^-} + 2\sqrt{3}\mu_{\Sigma^0\Lambda} + 5\mu_p + 8\mu_n = 0, \quad (2\%)$$
$$\mu_{\Omega^-} + 4\mu_{\Xi^0} - 3\mu_{\Xi^-} + 8\mu_{\Sigma^+} + 5\mu_{\Sigma^-} - 3\mu_p + \mu_n = 0, \quad (8\%)$$
$$4\mu_{\Xi^0} - \mu_{\Sigma^+} - \mu_{\Sigma^-} + 4\sqrt{3}\mu_{\Sigma^0\Lambda} - 6\mu_\Lambda + 4\mu_n = 0, \quad (6\%). \tag{30}$$

The last of these relations was noted in ref. [12], and is valid to $O(m_s)$ independent of the $1/N$ expansion. The average deviation of a fit which treats all of the counterterm couplings as free parameters is 0.08 μ_N, the same as the fit including only the $O(N m_s^{1/2})$ terms.

4. Predictions

We can use the results obtained above to predict the unmeasured decuplet magnetic moments including all contributions up to $O(N\epsilon)$. These may be measured in the future at CEBAF.

Up until now, the fits were used only to give a rough idea of how well the expansion works. We now use the fit to the magnetic moment data to make predictions, so we give some details on the fit and the treatment of errors. We add a theoretical uncertainty of 0.1 μ_N in quadrature with the experimental error to obtain the error on the individual magnetic moments, and then perform a χ^2 fit. This theoretical uncertainty is approximately the average deviation of the fit when corrections up to $O(N\epsilon)$ are included, and is consistent with the expectations of dimensional analysis: the largest contributions not included are $m_s^{3/2}$ nonanalytic terms

$$\delta\mu \sim \frac{M_K^3 M_N}{16\pi f^2 \Lambda^2}\mu_N \sim 0.2\ \mu_N. \tag{31}$$

We then obtain the predictions (in Bohr magnetons)

$$\begin{aligned}
\mu_{\Delta^{++}} &= 5.9 \pm 0.4, \\
\mu_{\Delta^+} = -\mu_{\Delta^-} &= 2.9 \pm 0.2, \\
\mu_{\Sigma^{*+}} &= 3.3 \pm 0.2, \\
\mu_{\Sigma^{*0}} &= 0.3 \pm 0.1, \\
\mu_{\Sigma^{*-}} &= -2.8 \pm 0.3, \\
\mu_{\Xi^{*0}} &= 0.65 \pm 0.2, \\
\mu_{\Xi^{*-}} &= -2.3 \pm 0.15,
\end{aligned} \tag{32}$$

The error quoted is purely the formal error from the fit. Note that in the limit of exact $SU(3)$ flavor symmetry, the decuplet magnetic moments are proportional to their charges. Eq. (32) gives definite predictions for the pattern of $SU(3)$ violation which would be very interesting to test.

We make no prediction for the Δ^0 magnetic moment. The leading contribution to this magnetic moment comes from terms such as

$$\delta\mathcal{L} = \frac{eb_4}{N\Lambda^2}\,\mathrm{tr}(m\mathcal{Q})\,v^\mu \epsilon_{\mu\nu\lambda\rho}F^{\nu\lambda}(\mathcal{B}|\{\sigma^\rho\}|\mathcal{B}), \tag{33}$$

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where $b_4 \sim 1$. (The reason for the factor of $1/N$ in the coefficient is that the quark diagrams which contribute to this term contain a fermion loop; see refs. [4][5].) This gives a contribution to the magnetic moments $O(1/N) = O(N\epsilon^2)$, which is higher order than the terms we are keeping.

Decuplet magnetic moments were also considered in a chiral expansion in ref. [13]. This paper computes $O(m_s^{1/2})$ and $O(m_s \ln m_s)$ $SU(3)$-violating contributions to the decuplet magnetic moments without making use of the $1/N$ expansion. Their predictions differ considerably from ours: for example, their predicted values for $\mu_{\Sigma^{*0}}$ and $\mu_{\Xi^{*0}}$ are significantly smaller than ours. It is worth noting that they do not include $O(m_s)$ counterterm contributions, which are not expected to be significantly smaller than the $O(m_s \ln m_s)$ which they compute, whereas the largest terms which are omitted in our analysis are suppressed by a *power* of the expansion parameter. We hope that experiment will be able to decide between these different predictions in the future.

The radiative decays of decuplet baryons also receive contributions from the same terms that give rise to the magnetic moments. However, the momentum transfer for the process $T \to B\gamma$ is

$$|\vec{q}| \sim M_T - M_B \sim \frac{\Lambda}{N}, \tag{34}$$

and the loop graph in fig. 1a has nontrivial dependence on $|\vec{q}|/(M_T - M_B) \sim 1$. This makes the coefficients of the effective operators in the expansion different from the ones appearing in the expansion of the magnetic moments. In principle, this difference is computable, but it depends on the value of the axial coupling g, which we know is not well determined. We will return to these issues in a future publication.
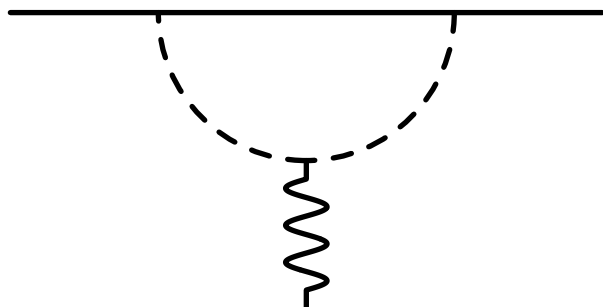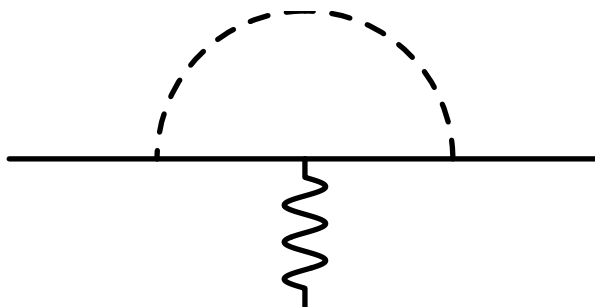
5. Acknowledgements

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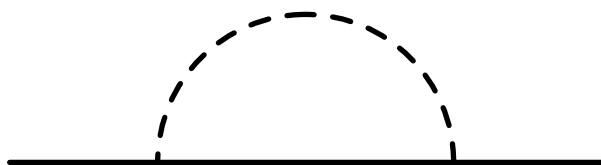
Figure Captions

Fig. 1. Feynman graphs giving rise to nonanalytic corrections to the baryon magnetic moments. Fig. 1c is wavefunction renormalization; fig. 1e does not contribute to the magnetic moments.
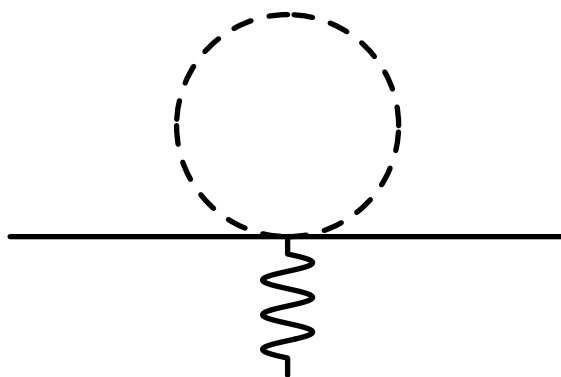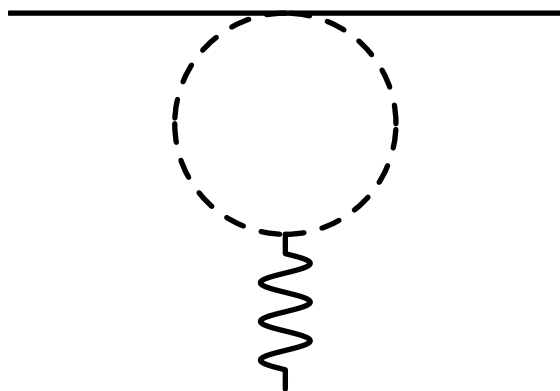
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